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Measuring complexity

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Abstract

Complexity is heterogenous, involving nonlinearity, self-organisation, diversity, adaptive behaviour, among other things. It is therefore obviously worth asking whether purported measures of complexity measure aggregate phenomena, or individual aspects of complexity and if so which. This paper uses a recently developed rigorous framework for understanding complexity to answer this question about measurement. The approach is two-fold: find measures of individual aspects of complexity on the one hand, and explain measures of complexity on the other. We illustrate the conceptual framework of complexity science and how it links the foundations to the practised science with examples from different scientific fields and of various aspects of complexity. Furthermore, we analyse a selection of purported measures of complexity that have found wide application and explain why and how they measure aspects of complexity. This work gives the reader a tool to take any existing measure of complexity and analyse it, and to take any feature of complexity and find the right measure for it.

1 Introduction

Measures of complexity have been proposed for decades and keep being proposed. Their interpretation, however, is rarely obvious. Measurement is an important part of any natural science and more and more of social sciences, as well. For a measure to be useful, however, it is necessary to understand what it means. The big hurdle in this case is that the understanding of complexity has been developing in parallel, with full agreement on its phenomenology still missing. In recent work, we have introduced a conceptual framework for the phenomenon of complexity which covers all the well-known phenomena that are associated with complexity – self-organisation, nonlinearity etc – and explains their relation to each other and to the phenomenon of complexity (Ladyman & Wiesner 2019, 2020). Here we link these foundations of complexity to complexity science in practice. We show, for the first time, the relation between the mathematical and computational tools that are being used in the field and the phenomenology of complexity. In particular, we use the *features* of complexity, as identified in (Ladyman & Wiesner 2019, 2020), to select measures that quantify these features. Furthermore, we select a few well-known so-called *measures of complexity* to illustrate how the framework helps to understand what aspect of complexity they quantify.

The field of complexity science is relatively young, it has existed under this name for less than fifty years.¹ Complexity science has its origins in systems science, nonlinear dynamical systems theory, and cybernetics, which all have experienced their main

¹The first institute in its name was founded in 1984 in Santa Fe, U.S. (Pines 2019). For an instructive visualisation of the history of complexity science, see the the infographic (Castellani 2018).

developments between the 1940s and the 1970s. It is no coincidence that the rise of complexity science was contemporary with the rise of available computational power. The role of computation is so central to the study of complex systems because their many elements and the many interactions between them that lead to the emergent phenomena of self-organisation and others can most often only be investigated with the help of computer simulations. Today, there is effectively no complex system that is not studied with the help of computation. Hence, some of the features of complexity are explored with simulations rather than with single measures applied to data.

2 What is a complex system?

In recent work, we have developed a framework for understanding ‘complexity’ applicable across the natural and social sciences (Ladyman & Wiesner 2019, 2020). We have distilled a list of features that are exhibited by complex systems; some features are exhibited by all complex systems, some only by functional or living complex systems. We distinguish between conditions for complexity and products of complexity. In a nutshell, the products are the ‘emergent’ properties that arise because of the many disordered interactions between the many parts and the feedback from previous interactions in systems that are open to the environment in some way. The latter are the ‘conditions for complexity’ (numerosity of elements and interactions, disorder, feedback, non-equilibrium). Table 1 lists the features of complexity that were identified. Not all products are present in all complex systems. In particular, as mentioned above, some are only present in functional or living complex systems (robustness of function, adaptive behaviour, modularity, memory). This is true by definition of these properties. Examples of non-functional / non-living systems are the universe and many condensed-matter forming systems, in particular when they exhibit phase transitions.

There is an important distinction between the *order of* a complex systems and the *order produced by* a complex system. An example of order produced by a complex system is a snowflake; the complex system that produced it is a cloud and the weather system it is part of. Real complex systems are always dynamic, but they often produce static order. Another example of a complex system that produces order is a honey bee hive; the *order of* the hive is the self-organised patterns of labour distribution for example; the (static) order *produced by* the hive are honey combs with their intricate hexagonal structure. In short, a complex system is a system that exhibits all of the conditions for complexity and at least one of the products emerging from the conditions. Here, we will not discuss these features much further. For details, see (Ladyman & Wiesner 2019), and for an in depth discussion, see (Ladyman & Wiesner 2020).

3 How to measure complexity

Measurement is imperative in the natural sciences. It is therefore not surprising that there has been an interest in measuring complexity ever since the beginning of complexity science in the 1980s.² But we agree with Murray Gell-Mann who noted, that “a variety of different measures would be required to capture all our intuitive ideas about what is meant by complexity” (Gell-Mann 1995). While we agree that measurement is, indeed, imperative, because complexity is so heterogeneous, it is not measurable except as an aggregate phenomenon or in individual aspects. In the light of the features of complexity presented in Section 2, such an aggregate measure can be designed as an aggregate of measures of features of complexity. There are no doubt plenty valid variations one could use and we will not propose one here. Instead, we provide the foundations for such

²A “nonexhaustive list” was compiled by Lloyd (2001).

Table 1. The features of complexity, as identified in (Ladyman & Wiesner 2019, 2020) where they have been grouped into ‘conditions for complexity’ and ‘products of complexity’.

Conditions of complexity	Numerosity of elements Numerosity of interactions Disorder non-equilibrium (openness) Feedback
Products of complexity	Nonlinearity Self-organisation Robustness of order Nestedness Robustness of function Adaptive behaviour Modularity Memory

discussions by illustrating (1) which measures exist and are in use to quantify features of complexity (Section 3.1), and (2) which feature of complexity is quantified by existing purported measures of complexity (Section 4). We will not present an exhaustive list of measures but give a sufficient guide for anyone to extend the list.

3.1 Measuring features of complexity

We now revisit the table of features (Tab. 1) and give a few examples of measures for some of the features. There are many others, and the best way to measure a certain feature will likely depend on the system that is under study. An in-depth discussion of this can be found in (Ladyman & Wiesner 2020).

3.1.1 Measuring disorder and diversity

Disorder in the dynamic of a complex system is a generating force. For example, the collective coordination of human groups can be improved by inserting a few autonomous agents acting with small levels of random noise (Shirado & Christakis 2017). The standard function from statistics to measure disorder is the *variance*: Consider a random variable X over numeric events $x \in \mathcal{X}$ in some alphabet \mathcal{X} with probability $P(x) := \Pr(X = x)$. the variance of X is defined as

$$\text{Var } X := \langle (X - \langle X \rangle)^2 \rangle . \quad (1)$$

In physics, the more commonly used function is the ‘standard deviation’ σ which is the square root of the variance, $\sigma = \sqrt{\text{Var } X}$. The variance quantifies the average deviation from the mean.

In complex systems, events are often non-numeric. Here the *Shannon entropy* comes in very handy.³ The Shannon entropy $H(X)$ of a random variable X (of numeric or non-numeric events) is a function of the probability distribution alone, and it is defined as

$$H(X) := - \sum_{x \in \mathcal{X}} P(x) \log_2 P(x) . \quad (2)$$

³A comprehensive introduction to information theory is provided in the textbook by Cover & Thomas (2012).

The Shannon entropy is maximised by the uniform probability distribution and minimised by the delta distribution of $P(x) = 1$ for some $x \in \mathcal{X}$ and $P(x) = 0$ otherwise. For a collections of n random variables $X_1 X_2 \dots X_n =: X_1^n$ with joint probability distribution $P(X_1 X_2 \dots X_n)$, the Shannon entropy is defined analogously:

$$H(X_1 X_2 \dots X_k) = - \sum_{x_i \in \mathcal{X}^n} \Pr(X_1^n = x_i) \log_2 \Pr(X_1^n = x_i) . \quad (3)$$

An example of the Shannon entropy as a measure of disorder in a complex system is the study by Wiesner et al. (2018). The authors used gene expression data of haematopoietic stem cells over the time course of their initial differentiation. Single-cell technology allowed the measurement of individual cells' gene activity and the extraction of probability distributions over these. A set of genes, known to be relevant for stem cell differentiation, was measured for a set of cells over a sequence of time steps. From this, the probability of a particular gene being switched on or off, resolved in time, could be extracted. The authors computed the Shannon entropy for the set of genes resolved in time. The average entropy values indicated a clear increase toward the point of differentiation. This was a novel observation and corroborated ideas of noise being relevant in transitions between stable states of complex systems (Enver et al. 1998).

Disorder is often relevant in temporal measurement sequences. The *Shannon entropy rate* is a measure of disorder. Given a sequence of random variables, $X_1 X_2 \dots X_n$, and their joint Shannon entropy $H(X_1 X_2 \dots X_n)$, the Shannon entropy rate is defined as

$$h_n := \frac{1}{n} H(X_1 X_2 \dots X_n) . \quad (4)$$

The limit $n \rightarrow \infty$ plays an important role in dynamical systems theory (Jost 2006a). Another definition of the entropy rate is $H(X_n | X_1 X_2 \dots X_{n-1})$, where $H(\cdot | \cdot)$ is the conditional entropy. As $n \rightarrow \infty$, these two entropy rates converge to the same limit. The latter expression makes it more apparent that the entropy rate quantifies the amount of uncertainty about the measurement at time n when the system has already been observed for $n - 1$ time steps. The more ordered the system is, the less the uncertainty about its dynamic at the next time step.

The Shannon entropy rate has been used as a measure of disorder, for example, in the study of van Steveninck et al. (1997) on the neural activities of flies. The authors measured the neural activity of flies while, during some recordings, exposing the flies to controlled regular visual stimuli. From the neural-activity recordings, probability distributions were extracted over sequences of on-off activity for individual motion-sensitive neural regions. From these probability distributions, the joint entropies were computed and, finally, the entropy rate (Eq. 4). The authors found that the entropy rate varied significantly between recordings with and without the visual stimulus. When the flies were exposed to a visual stimulus, the entropy rate was much lower than when the flies were not exposed to any controlled stimulus. The increase in Shannon entropy rate upon the absence of a regular visual stimulus was interpreted as an increase in the disorder of the flies neural activity pattern.

Disorder and diversity are related phenomena in the sense that they can often be quantified by the same mathematical tools. Diversity is often a stabilising force in complex systems, increasing robustness and resilience (Page 2010). For example, it has been shown that a bee hive colony with a diverse genome has higher survival rates than a colony with a more homogeneous genome (Mattila & Seeley 2007). The Shannon entropy is used to measure both disorder and diversity. In particular, the exponentiated value of the Shannon entropy, $e^{H(X)}$, is a standard measure of species diversity (Jost 2006b).

3.1.2 Measuring order

Order in complex systems comes in two forms. As mentioned above, there is the *order of* a complex systems and the *order produced by* a complex system. An example of the former is the coherent movement of a flock of birds; an example of the latter are the honey combs produced by honey bees. In general, order occurs in the form of either spatial correlations between the elements that constitute the system or in the form of temporal correlations in their behaviour. A complex system can exhibit, and usually does exhibit, both spatial and temporal order. Self-organisation is prominent in the discussion of complex systems because it usually is very visible. It is therefore no coincidence that many purported measures of complexity are measures of order.⁴ A common way of measuring order in complex systems is to use the covariance from statistics or the the correlation functions from statistical physics. Consider two numeric random variables X and Y . The *covariance* is defined as

$$\text{cov}(X, Y) = \langle XY \rangle - \langle X \rangle \langle Y \rangle . \quad (5)$$

In statistical physics, the convention is to not subtract the product of the marginal expectation values and, instead, use the first term only:

$$C_{X,Y} := \langle XY \rangle . \quad (6)$$

This is called the correlation function. Correlation functions for time correlations, for example, can be written as

$$C_X(t) := \langle X(t)X(t_0) \rangle . \quad (7)$$

There are other standard statistical measures of correlation that are widely used, such as the Pearson correlation (which is the covariance divided by the product of the standard deviations).

For non-numeric random variables, again, information theory comes in handy. The mutual information (Cover & Thomas 2012) is a popular measure of correlation, not least because it captures linear as well as nonlinear correlations.

An example of the covariance being used as a measure of order in a complex system is the study of flocks of starlings in the sky of Rome by Bialek et al. (2012). The authors recorded videos of large flocks of starlings in coherent collective movement and extracted the flight paths of individual birds from these vidoes. Each bird’s flight direction was represented as a random variable, from which the correlation functions (Eq. 6) of all pairs of birds was computed. This set of correlations was the input to a simulation of flocking behaviour such that the simulated flocks had the same amount of correlation on average as the real starlings, with no further parameters used to tune the behaviour. The purpose of measuring the correlations was not for the sake of measuring order alone but a proof of concept that correlated movements such as that of flocks of starlings can be generated by pairwise correlations alone, and that no triple or higher order correlations are necessary. Hence, self-organisation in complex systems, measured by measuring the order that is generated, does not necessarily require a central controller or any higher-order communication between the self-organising entities. A similar study was performed on network data of cultured cortical neurons by Schneidman et al. (2006).

3.2 Measuring other features of complexity

We will briefly mention a few other measures of features of complexity. Nonlinearity has more than one meaning in complexity. There is the association of nonlinearity

⁴The much stated “Complexity lies between order and disorder” is a confusion of the fact that in complex systems, both order and disorder are always present (Ladyman et al. 2013).

with correlations (MacKay 2008), and the nonlinearity of power laws, of course (see, eg, (Clauset et al. 2009)). Nonlinearity is also found in the presence of critical transitions and tipping points (Scheffer 2009). A measure of modularity for complex networks, for example, has been introduced by Newman & Girvan (2004). For an in-depth discussion, see (Ladyman & Wiesner 2020).

4 Examples of ‘complexity measures’ and their interpretation

In the following, we choose two areas of complexity science – neuroscience and economics – to illustrate the use of separating the aggregate phenomenon of complexity into its features.

4.1 Economic complexity

Tacchella et al. (2012) introduced a measure of economic complexity which is two measures in one: ‘country fitness’ and ‘product complexity’. The two measures are functions of a matrix, the so-called country-product matrix, M , in which each row a country and each column is a product for export. A matrix entry M_{cp} equals one when country c exports product p , and it equals zero otherwise. In a recursive algorithm, a country’s fitness F_c and a product’s complexity Q_p are determined by the following equations:

$$F_c^{(n)} = \sum_p M_{cp} Q_p^{(n-1)}, \quad (8)$$

$$Q_p^{(n)} = \frac{1}{\sum_c M_{cp} \frac{1}{F_c^{(n-1)}}}. \quad (9)$$

The underlying idea is that products that are exported by few countries are ‘complex’, and that countries that export many different products are fitter than those that export only few products. Based on these two measures, the authors are able to provide new insights into patterns of production and international export. The underlying idea makes it clear that ‘country fitness’ is a measure of diversity of a country’s export portfolio. ‘Product complexity’ is a derived quantity from the country that exports it. One might argue that a ‘complex’ product is one that requires a diverse production infrastructure. In this sense, both ‘country fitness’ and ‘product complexity’ are measures of diversity.

A few years previous to this, Hidalgo & Hausmann (2009) introduced a measure they call ‘economic complexity’. It, too, is two measures in one: an ‘economic complexity index’ (ECI) and a ‘product complexity index’ (PCI). Originally, these measures were interpreted as quantifying diversity. But the ‘economic complexity’ has been analysed by Mealy et al. (2019) and found to be equivalent to a spectral clustering algorithm that partitions a similarity graph into two parts.

4.2 Brain complexity

The complexity by Tononi et al. (1994) is a much cited complexity measure, also called ‘TSE complexity’. To our knowledge, it is not being used in practice. Its set-up is as follows. Assume a collection of n components⁵ that can be either active or inactive (or possibly be in one of more than two states). For all these components, a recording exists of

⁵In Tononi et al. (1994), the components are neural groups, but for the purpose of the mathematics of this measure, their nature is irrelevant.

their activity over time. It is assumed that the recording is such that reliable probabilities for the activities of individual and collections of components can be extracted. This yields the Shannon entropy of brain region j with m different activity states:

$$H(X_j) = - \sum_{i=1}^m \Pr(X_j = x_i) \log_2 \Pr(X_j = x_i) . \quad (10)$$

For a collections of k components $X_j^k := X_{j_1} X_{j_2} \dots X_{j_k}$ with joint probability distribution $P(X_j^k)$, the Shannon entropy can be written as

$$H(X_j^k) = - \sum_{i=1}^{m^k} \Pr(X_j^k = x_i) \log_2 \Pr(X_j^k = x_i) . \quad (11)$$

The average over all n/k collections of k components is then

$$\langle H(X_j^k) \rangle = \sum_{j=1}^{n/k} \Pr(X_j^k) H(X_j^k) = \sum_{j=1}^{n/k} H(X_j^k) , \quad (12)$$

since the probability distribution over collections is assumed to be uniform. From information theory we know that (Cover & Thomas 2012)

$$H(X_j^k) \leq \sum_{i=1}^k H(X_{j_i}) , \quad (13)$$

and that any deviation from equality is caused by correlations between the components. Tononi et al. (1994) have, therefore, in their definition of the complexity measure $C_N(X)$ (the subscript N stands for ‘neural’), made the natural choice of subtracting one from the other in the following way:

$$C_N(X) := \sum_{k=1}^n \langle H(X_j^k) \rangle - \frac{k}{n} H(X) , \quad (14)$$

where $H(X)$ is the joint entropy of the entire system.⁶

We can see from this brief summary of the TSE complexity definition that C_N is a measure of order. For maximally uncorrelated activity across all collections of neurons this measure is zero. This can be seen as follows: assume a collection of components that, individually, have maximum entropy, $H(X_j) = \log_2 m$. Inequality 13 turns into an equality, and we get zero for all terms in the sum of $C_N(X)$. As soon as there is a level of synchronisation in the activity across components, C_N will be greater than zero. It is not obvious, which probability distribution maximises C_N and whether it is unique. As the authors state themselves: “ $C_N(X)$ is high, when, on the average, the mutual information between any subset of the system and its complement is high.” This interpretation is not surprising since the mutual information is a well-known measure of correlation and, hence, of order.

The TSE complexity is not dissimilar from the predictive complexity (check reference!) (Bialek & Tishby 1999), effective complexity (Grassberger 1986), and the excess entropy (Crutchfield & Feldman 2003). These measures are measures designed for time sequences and capture all correlations across arbitrary lengths of time. The TSE complexity, on the other hand, captures all correlations across collections of arbitrary numbers of components in space.

⁶We use the notation of the original paper. It might be helpful to add that $H(X) \equiv H(X_1^n)$ since there is only one such collection of n components. The last term of the sum in Eq. 14 is zero by definition, and the sum might as well run from $k = 1$ to $k = n - 1$.

4.3 Complexity vs Chaos

In nonlinear dynamical systems theory quite a few measures are labelled ‘complexity measures’. What ‘complexity’ means in this context is non-trivial dynamics such as chaotic dynamics. Hence, some of the measures are designed to give high values for systems with chaotic dynamics. Two examples out of many are the ‘approximate entropy’ (Pincus 1991) and the ‘permutation entropy’ (Bandt & Pompe 2002). They illustrate that chaos and complexity are often conflated in the literature. We consider this an historical artefact due to the subjects having contemporary development.

5 Conclusions

The study of coupled human and natural systems is becoming ever more important. The science that is required to understand and predict issues such as climate change and migration is necessarily crossing many scientific borders. For scientific rigour mathematical tools become ever more important, including in the social sciences. A unifying framework for complexity facilitates this cross-disciplinary endeavour. Furthermore, it allows for quantifying approaches to issues such as economic growth. However, there is confusion about the nature of complexity even within the natural scientific disciplines. The analysis in terms of complexity features developed in (Ladyman & Wiesner 2019, 2020) and the present paper’s systematic analysis of mathematical measures of these features is intended to bring clarity to the discussion of complexity across all disciplines. We gave examples from economics and neuroscience of purported complexity measures that are prominent in the literature. We clarified the interpretation for some of these and incorporated all interpretations into the framework of features of complexity. With the list of features of complexity at hand, any measure of complexity can be analysed and its interpretation will likely be one of the ten features of complexity, or an aggregation of them. At the same time, with the above analysis, features of complexity that are important for a given system can now be identified and measures for them can be designed.

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